Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 2

Specimen paper

2 hours

Instructions to candidates

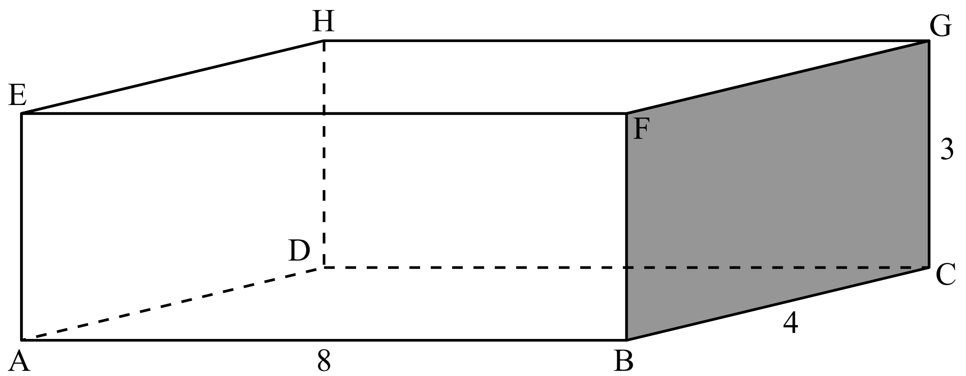
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 15]

The Happy Straw Company manufactures drinking straws.

The straws are packaged in small closed rectangular boxes, each with length $8\,\text{cm}$, width $4\,\text{cm}$ and height $3\,\text{cm}$. The information is shown in the diagram.



(a) Calculate the surface area of the box in cm^2. [2]

(b) Calculate the length AG. [2]

Each week, the Happy Straw Company sells x boxes of straws. It is known that $\frac{\mathrm{d}P}{\mathrm{d}x} = -2x + 220$, $x \geq 0$, where P is the weekly profit, in dollars, from the sale of x thousand boxes.

(c) Find the number of boxes that should be sold each week to maximize the profit. [3]

The profit from the sale of $20\,000$ boxes is $\$1700$.

(d) Find $P(x)$. [5]

(e) Find the least number of boxes which must be sold each week in order to make a profit. [3]

2. [Maximum mark: 12]

Slugworth Candy Company sell a variety pack of colourful, shaped sweets.

The sweets are produced such that 80% are star shaped and 20% are shaped like a crescent moon. It is known that 10% of the stars and 30% of the crescent moons are coloured yellow.

(a) A sweet is selected at random.

(i) Find the probability that the sweet is yellow.

(ii) Given that the sweet is yellow, find the probability it is star shaped. [4]

According to manufacturer specifications, the colours in each variety pack should be distributed as follows.

Colour	Brown	Red	Green	Orange	Yellow	Purple
Percentage (%)	15	25	20	20	10	10

Mr Slugworth opens a pack of 80 sweets and records the frequency of each colour.

Colour	Brown	Red	Green	Orange	Yellow	Purple
Observed Frequency	10	20	16	18	12	4

To investigate if the sample is consistent with manufacturer specifications, Mr Slugworth conducts a χ^2 goodness of fit test. The test is carried out at a 5% significance level.

(b) Write down the null hypothesis for this test. [1]

(c) **Copy** and complete the following table in your answer booklet. [2]

Colour	Brown	Red	Green	Orange	Yellow	Purple
Expected Frequency						

(d) Write down the number of degrees of freedom. [1]

(e) Find the p-value for the test. [2]

(f) State the conclusion of the test. Give a reason for your answer. [2]

3. [Maximum mark: 18]

In this question, give all answers to two decimal places.

Bryan decides to purchase a new car with a price of €$14\,000$, but cannot afford the full amount. The car dealership offers two options to finance a loan.

Finance option A:

A 6 year loan at a nominal annual interest rate of 14% **compounded quarterly**. No deposit required and repayments are made each quarter.

(a) (i) Find the repayment made each quarter.

(ii) Find the total amount paid for the car.

(iii) Find the interest paid on the loan. [7]

Finance option B:

A 6 year loan at a nominal annual interest rate of $r\,\%$ **compounded monthly**. Terms of the loan require a 10% deposit and monthly repayments of €250.

(b) (i) Find the amount to be borrowed for this option.

(ii) Find the annual interest rate, r. [5]

(c) State which option Bryan should choose. Justify your answer. [2]

Bryan chooses option B. The car dealership invests the money Bryan pays as soon as they receive it.

(d) If they invest it in an account paying 0.4% interest per month and inflation is 0.1% per month, calculate the real amount of money the car dealership has received by the end of the 6 year period. [4]

4. [Maximum mark: 14]

An aircraft's position is given by the coordinates (x, y, z), where x and y are the aircraft's displacement east and north of an airport, and z is the height of the aircraft above the ground. All displacements are given in kilometres.

The velocity of the aircraft is given as $\begin{pmatrix} -150 \\ -50 \\ -20 \end{pmatrix} \text{km h}^{-1}$.

At $13:00$ it is detected at a position $30\,\text{km}$ east and $10\,\text{km}$ north of the airport, and at a height of $5\,\text{km}$. Let t be the length of time in hours from $13:00$.

(a) Write down a vector equation for the displacement, $\boldsymbol{r}$ of the aircraft in terms of t. [2]

(b) If the aircraft continued to fly with the velocity given

(i) verify that it would pass directly over the airport;

(ii) state the height of the aircraft at this point;

(iii) find the time at which it would fly directly over the airport. [4]

When the aircraft is $4\,\text{km}$ above the ground it continues to fly on the same bearing but adjusts the angle of its descent so that it will land at the point $(0, 0, 0)$.

(c) (i) Find the time at which the aircraft is $4\,\text{km}$ above the ground.

(ii) Find the direct distance of the aircraft from the airport at this point. [5]

(d) Given that the velocity of the aircraft, after the adjustment of the angle of descent, is $\begin{pmatrix} -150 \\ -50 \\ a \end{pmatrix} \text{km h}^{-1}$, find the value of a. [3]

5. [Maximum mark: 17]

The following table shows the costs in US dollars (US$) of direct flights between six cities. Blank cells indicate no direct flights. The rows represent the departure cities. The columns represent the destination cities.

		Destination city					
		A	**B**	**C**	**D**	**E**	**F**
Departure city	**A**		90	150			
	B	90		80	70	140	
	C	150	80				
	D		70			100	180
	E		140		100		210
	F				180	210	

(a) Show the direct flights between the cities as a graph. [2]

(b) Write down the adjacency matrix for this graph. [2]

(c) Using your answer to part (b), find the number of different ways to travel from and return to city A in exactly 6 flights. [2]

(d) State whether or not it is possible to travel from and return to city A in exactly 6 flights, having visited each of the other 5 cities exactly once. Justify your answer. [2]

The following table shows the least cost to travel between the cities.

		Destination city					
		A	**B**	**C**	**D**	**E**	**F**
Departure city	**A**	0	90	150	160	a	b
	B	90	0	80	70	140	250
	C	150	80	0	150	220	330
	D	160	70	150	0	100	180
	E	a	140	220	100	0	210
	F	b	250	330	180	210	0

(e) Find the values of a and b. [2]

A travelling salesman has to visit each of the cities, starting and finishing at city A.

(f) Use the nearest neighbour algorithm to find an upper bound for the cost of the trip. [3]

(g) By deleting vertex A, use the deleted vertex algorithm to find a lower bound for the cost of the trip. [4]

6. [Maximum mark: 14]

A city has two cable companies, X and Y. Each year 20% of the customers using company X move to company Y and 10% of the customers using company Y move to company X. All additional losses and gains of customers by the companies may be ignored.

(a) Write down a transition matrix $\boldsymbol{T}$ representing the movements between the two companies in a particular year. [2]

(b) Find the eigenvalues and corresponding eigenvectors of $\boldsymbol{T}$. [4]

(c) Hence write down matrices $\boldsymbol{P}$ and $\boldsymbol{D}$ such that $\boldsymbol{T} = \boldsymbol{PDP}^{-1}$. [2]

Initially company X and company Y both have 1200 customers.

(d) Find an expression for the number of customers company X has after n years, where $n \in \mathbb{N}$. [5]

(e) Hence write down the number of customers that company X can expect to have in the long term. [1]

7. [Maximum mark: 20]

An object is placed into the top of a long vertical tube, filled with a thick viscous fluid, at time $t = 0$.

Initially it is thought that the resistance of the fluid would be proportional to the velocity of the object. The following model was proposed, where the object's displacement, x, from the top of the tube, measured in metres, is given by the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 9.81 - 0.9\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right).$$

(a) By substituting $v = \frac{\mathrm{d}x}{\mathrm{d}t}$ into the equation, find an expression for the velocity of the particle at time t. Give your answer in the form $v = f(t)$. [7]

The maximum velocity approached by the object as it falls is known as the terminal velocity.

(b) From your solution to part (a), or otherwise, find the terminal velocity of the object predicted by this model. [2]

An experiment is performed in which the object is placed in the fluid on a number of occasions and its terminal velocity recorded. It is found that the terminal velocity was consistently smaller than that predicted by the model used. It was suggested that the resistance to motion is actually proportional to the velocity squared and so the following model was set up.

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 9.81 - 0.9\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2$$

(c) Write down the differential equation as a system of first order differential equations. [2]

(d) Use Euler's method, with a step length of 0.2, to find the displacement and velocity of the object when $t = 0.6$. [4]

(e) By repeated application of Euler's method, find an approximation for the terminal velocity, to five significant figures. [1]

At terminal velocity the acceleration of an object is equal to zero.

(f) Use the differential equation to find the terminal velocity for the object. [2]

(g) Use your answers to parts (d), (e) and (f) to comment on the accuracy of the Euler approximation to this model. [2]